

Euphrates Pollution Threats for Human and Fishery Wealth Survival in Al Hindyia City by using 2D Mathematical Dispersion-advection Modeling

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Abstract:

A brief study is performed including mathematical derivation, conceptualization and computer programming to develop 2D comprehensive mathematical model to dealing with any environmental problems especially that concerning pollution accident. The model is capable to be used for solving a point, line and area sources. The current pollution process in Al Hindyia city which is located on Euphrates River proves the model is efficient tool to deal with any environmental pollution which may be occurred in stilling and moving surface water. The results shows that the point at which the waste water concentration reaches a zero value is occurred at 555m D/S the disposing point into the Euphrates River.

Keywords: Zero-Concentration Point, Diffusivity, Nstep

Introduction

Nowadays most of the worldwide has been exposing to severe pollution in air, surface water and groundwater pollution. The carelessness of human in his environment exaggerates the sources and even types of pollutions. Organics, inorganics, and radioactive among toxic and harmful minerals that may cause contamination threads for human health. Euphrates River is one of most surface boundary that may endure the municipal pollution in Al Hindyia City.

The general one dimensional non-steady partial differential equations that govern the changes in pollutant and dissolved oxygen concentration as indicated by Chapara (1997) are:-

$$\frac{\partial(AP)}{\partial t} = D_x \frac{\partial^2(AP)}{\partial x^2} - \frac{\partial(vAP)}{\partial x} + qH(x) \quad \text{subjected to } (x < L < \infty), t > 0$$

Pimpunchat et al (2008) have developed a mathematical model for Tha Chin River pollution comprising to investigate the effect of aeration on the degradation of pollutant. The disposing pollutant is (0.06 kg/ m day). The river is ecologically dead. From Scenario 1, the fish survival constraint Tha Chin river is $q < 0.015$. However, the oxygen level fortunately remains above the critical value of 30% of the saturated oxygen. Saleem et al (2008) developed a model to predict the pollutant concentration of a river at al Dywania. The model is run for $D_p = 0$ and $D_x = 0$. It is found that the measured and simulated pollution concentrations and dissolved oxygen at the critical value and identical for of AL-Diwaniya city.

Ramalinga and Sreenivasa (2013) presented a mathematical model for river pollution and evaluate the freshening effects on the humiliation of toxin The model comprises a coupled diffusion- advection process for toxin and dissolved oxygen concentrations. A steady-state case in one spatial dimension. A case study of pollution is undertaken krishna River in India.

A wide range of surface water pollution modelling, some models adopt a software, and other adopt direct analytical solutions. In the current modeling technique, a numerical 2D dispersion-advection model is developed to analyze Euphrates River pollution.

Case Study

A geographic location of Al Hindyia City between three parallel surface boundaries namely as, Euphrates River, Western and Eastern streams. The morphology layout of these hydrologic boundaries certainly do not let the population easily drain their waste municipal water far away from the city center.

Correspondingly, the habitants customize Longley disposing their municipal waste water in Euphrates River.

Research Significance & Challenges

Euphrates River, in recent decades has been exposed to a toxic hazards of contamination resulting from a careless daily human activity and bad disposing municipal waste material in the river. Accordingly, becomes most important to assess the degradation degree of the river pollution and its effects on human health and fishery survival within the river reach of Al Hindyia City shown in Fig.(1).

Research Objectives

The goals of research are:-

- Solving a diffusion-advection partial differential equation numerically.
- Development a 2D model for simulating a concentrations in a surface water flow.
- To assign the hazardous pollutants of Euphrates River reach in Al hindyia City by referring to World Health Organization Specifications and recommendations.
- Adopting the optimum solutions for remediating and perpetuating a good river environment outlooks.



Fig.(1) Al-Hnidya City and Euphrates River Reach

Mathematical Background

The general form of 3D advection-dispersion equation is:-

$$\frac{\partial c}{\partial t} = \nabla(D\nabla c) - \nabla \cdot (vc) + Q \quad \dots\dots\dots (1)$$

Where:

$\frac{\partial c}{\partial t}$ represent the temporal variation of chemical species concentration, the term $\nabla(D\nabla c)$ describes the variation diffusivity of the chemicals within the media, $\nabla \cdot (vc)$ quantitates the advection of the chemical species, and finally Q is the sink source discharge of the chemicals within media or in other words the creation or destruction of the quantity and how the molecule can be created or destroyed by chemical reactions. Briefly, concentration c for each discretized location should be solved by simultaneous differential equations.

Stationary advection-dispersion equ. describes the steady state system behavior that $\frac{\partial c}{\partial t} = 0$. Accordingly Equ.(1) becomes:

$$0 = \nabla(D\nabla c) - \nabla \cdot (vc) + Q \quad \dots\dots\dots (2)$$

The expansion form of transient Eq.(1) in Cartesian system is as:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_y \frac{\partial c}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_z \frac{\partial c}{\partial z} \right) - \frac{\partial v_x c}{\partial x} - \frac{\partial v_y c}{\partial y} - \frac{\partial v_z c}{\partial z} + Q \quad \dots\dots\dots (3)$$

Where:- c is the pollutant concentration, D_x, D_y and D_z (m^2/day dispersion coefficient in x, y and z directions), Q is the pollutant discharge (m^3/day) and t is the differential time (day)

D_x, D_y and D_z are considered to constant in x, y and z directions respectively therefore Equ. (3) becomes

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2} - \frac{\partial v_x c}{\partial x} - \frac{\partial v_y c}{\partial y} - \frac{\partial v_z c}{\partial z} + Q \quad \dots\dots\dots (3)$$

Rivers Modeling Consideration and Simplifications

Usually, the surface runoff in rivers and streams is considered to be one-dimensional flow but in really there is a local traverse flow in other perpendicular directions. In this analysis in order to proceed the solution of Equ.(3) two assumptions should be taken; they are:-

- 1- The media is considered non-homogenous isotropic accordingly the dispersivity is equal in x, y and y directions therefore $D_x = D_y = D_z = D$ ($\frac{m^2}{day}$).
- 2- Because of non-homogeneity, the average runoff velocity in x direction along the Euphrates River $v_x = m/day$ whereas traverse velocities $v_y = v_z = 0$ for one dimensional flow. Subsequently the transient form of the governing Equ.(3) becomes:-

$$\frac{\partial c}{\partial t} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right) - v_x \left(\frac{\partial c}{\partial x} \right) + Q \quad \dots\dots\dots(4)$$

General Numerical Solution

The discretization of 3D model domain requires to superimpose a square paper over a modeled area. The number of column is denoted by N_c , the number of Rows in y direction is denoted by N_R and the number of Layers in Z direction is denoted by N_L . In the current model the maximum $N_c = 30, N_R = 38$ and $N_L = 1$. Since the Number of $N_L = 1$ (*the depth of Euphrates < 25m*), therefore the model is reduced to 2D model. Accordingly Equ.(4) is becomes:-

$$\frac{\partial c}{\partial t} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) - v_x \left(\frac{\partial c}{\partial x} \right) + Q \quad \dots\dots\dots(4)$$

To solve Eq. (4) numerically by using the finite difference approximation of Taylor series expansion Briefly, the first and second derivatives in x -axis according to Taylor are as follows: Erwin Kreyszig (1972)

$$\frac{\partial c}{\partial x} = \frac{c_{i,j+1} - c_{i,j-1}}{2\Delta x}$$

$$\frac{\partial^2 c}{\partial x^2} = \frac{c_{i+1,j} - 2c_{i,j} + c_{i-1,j}}{\Delta x^2}$$

Similarly, the first and second derivatives in ordinate axis are as follows:

$$\frac{\partial c}{\partial y} = \frac{c_{i+1,j} - c_{i-1,j}}{2\Delta y}$$

$$\frac{\partial^2 c}{\partial y^2} = \frac{c_{i+1,j} - c_{i,j} + c_{i-1,j}}{\Delta y^2}$$

$$\frac{\partial c}{\partial t} = \frac{c_{i,j} - c_{0,j}}{\Delta t}$$

The numerical solution of Equ.(4) based on a finite difference approach which is required to discretize the domain into a number of nodes as shown in (Fig.(3) to construct a number of algebraic equations which should be solved simultaneously to find a numerical values of the variables for the node.

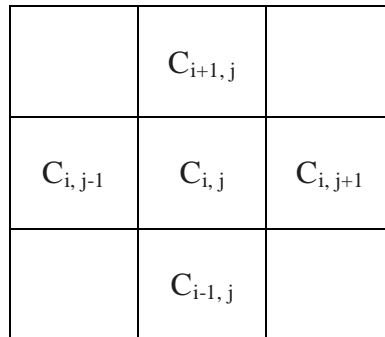


Fig.(3) Two dimensional explicit method of nodal points

The governing equation of the modal domain Equ. (4) may be of the form of recurrence Equ.(5) by using the central difference in x & y coordinates.

$$\frac{C_{i,j} - C_{0i,j}}{\Delta t} = D \left(\frac{C_{i,j-1} - 2C_{i,j} + C_{i,j+1}}{\Delta x^2} + \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{\Delta y^2} \right) - v_{i,j} \left(\frac{C_{i,j+1} - C_{i,j-1}}{2\Delta x} \right) + Q_{i,j} \dots\dots\dots(5)$$

Equ. (5) can be simplified to the form of Equ.(6) taking into account $\Delta x = \Delta y = \Delta$ for equal mesh discretization

$$\left(\frac{v_{i,j}\Delta}{D\Delta t} + 4 \right) C_{i,j} = + \left(1 - \frac{v_{i,j}}{2D} \right) C_{i,j+1} + C_{i+1,j} + C_{i-1,j} + \frac{Q_{i,j}}{D} \dots\dots\dots(6)$$

Rearrangement of Equ.(6) yields:-

$$\left(1 + \frac{v_{i,j}\Delta}{2D} \right) C_{i,j-1} + \left(1 - \frac{v_{i,j}\Delta}{2D} \right) C_{i,j+1} - \left(4 + \frac{v_{i,j}\Delta}{D\Delta t} \right) C_{i,j} + C_{i-1,j} + C_{i+1,j} = -\frac{\Delta^2}{D} \left(\frac{C_{0i,j}}{\Delta t} + Q_{i,j} \right)$$

If it is assumed that

$$AA_{i,j} = \left(1 + \frac{v_{i,j}\Delta}{2D} \right), BB_{i,j} = \left(1 - \frac{v_{i,j}\Delta}{2D} \right), CC_{i,j} = - \left(4 + \frac{v_{i,j}\Delta}{D\Delta t} \right), DD_{i,j} = 1, EE_{i,j} = 1 \text{ and}$$

$$DC_{i,j} = -\frac{\Delta^2}{D} \left(\frac{C_{0i,j}}{\Delta t} + Q_{i,j} \right)$$

Therefore by the substitutions of $AA_{i,j}, BB_{i,j}, CC_{i,j}, DD_{i,j}, EE_{i,j}$ and $DC_{i,j}$, Equ.(6) may be rearranged to give the compacted form of the algebraic governing equation of transient non-homogenous isotropic diffusion surface runoff of the river.

$$AA_{i,j}C_{i,j-1} + BB_{i,j}C_{i,j+1} + CC_{i,j}C_{i,j} + DD_{i,j}C_{i-1,j} + EE_{i,j}C_{i+1,j} = DC_{i,j} \dots\dots\dots(7)$$

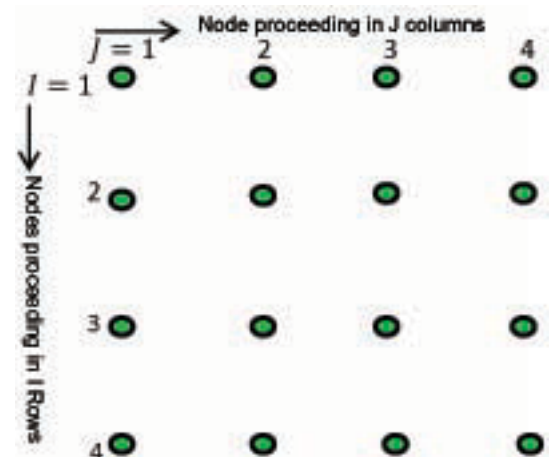
Where $AA_{i,j}, BB_{i,j}, CC_{i,j}, DD_{i,j}, EE_{i,j}$ and $DC_{i,j}$ are coefficients depend upon the physical properties of each mesh within model domain.

Technical solution by G arrays

The modified Gauss Seidel technique may be suitable

To estimate the concentration variable at each node sequentially G rows with time proceeding under any contamination physical Conditions, it is suitable to begin with Equ. (7). The C(I,J) is:-

$$CC_{i,j}C_{i,j} = DC_{i,j} - AA_{i,j}C_{i,j-1} - BB_{i,j}C_{i,j+1} - DD_{i,j}C_{i-1,j}$$



$$-EE_{i,j}C_{i+1,j} \dots\dots\dots(8)$$

Dividing both sides by $CC_{i,j}$, it becomes:-

$$C_{i,j} = \frac{DC_{i,j}}{CC_{i,j}} - \frac{AA_{i,j}}{CC_{i,j}} C_{i,j-1} - \frac{BB_{i,j}}{CC_{i,j}} C_{i,j+1} - \frac{DD_{i,j}}{CC_{i,j}} C_{i-1,j} - \frac{EE_{i,j}}{CC_{i,j}} C_{i+1,j} \dots\dots\dots(9)$$

Fig.(4) Four Nodes Example

If the G array is adapted by assuming that:

$$GC = \frac{DC_{i,j}}{CC_{i,j}}, GA = \frac{AA_{i,j}}{CC_{i,j}}, GB = \frac{BB_{i,j}}{CC_{i,j}}, GD = \frac{DD_{i,j}}{CC_{i,j}} \text{ and } GE = \frac{EE_{i,j}}{CC_{i,j}}, \text{ Eq.(9) is reduced to:}$$

$$C_{i,j} = GC - GA C_{i,j-1} - GB C_{i,j+1} - GD C_{i-1,j} - GE C_{i+1,j} \dots\dots\dots(10)$$

The determination of $C_{i,j}$ requires estimating the G arrays values in i, j nodal point and the preceded values of the surrounding nodes concentration variables namely as, $C_{i,j-1}$, $C_{i,j+1}$, $C_{i-1,j}$ and $C_{i+1,j}$.

Basic Simulation Programming

The simulation program is coded in Fortran Language by using the theoretical bases of equations (8, 9, and 10) for solving a set of simultaneous equations to operate with any consistent of units. Fig.(5) presents the basic steps of the program.

Simulation Setup & Modular Job

The first step in simulation job is the discretization of the model domain into a finite number of meshes. The number of meshes is depended upon the accuracy of the required output results. The more meshes the more accurate values. This is can be reached by superimposing a square paper on the scaled map of the domain. The number of meshes in x and y directions are coded as NC and NR respectively.

Base Map Implementation

It is defined as a number of meshes bounded the modal domain. All these cells should be traced in x and y direction (The diagonal tracing is not allowed) until the boundary of the domain is closed. The benefits of the base map technique are omission the outside output results of the simulated model domain and fixing the boundary conditions of the model domain. Fig.(6) presents the base map and the mesh design of typical model domain. The figure shows the enumeration of some cells surrounding the boundary that are completed to enclose the boundary of the modeled domain.

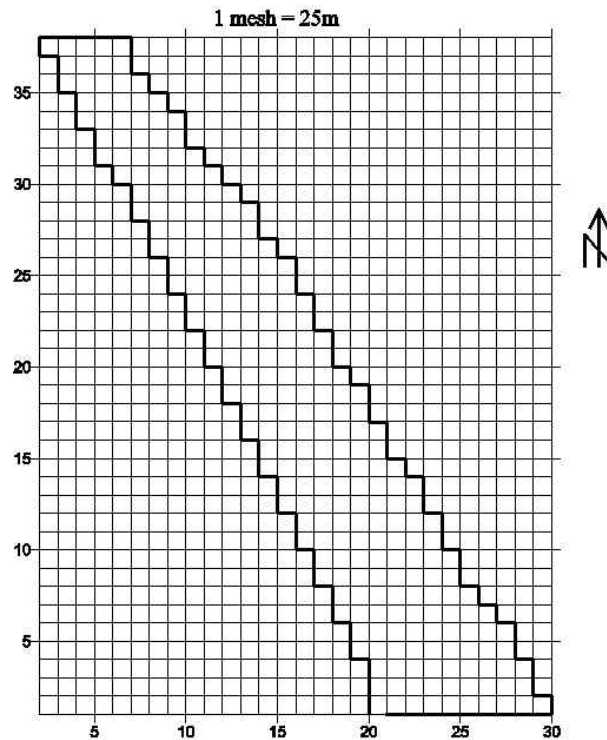


Fig.(6) Mesh Design &Base Map Implementation

Model Verification & Calibration

The time step namely as (Istep) is selected to be (0.1day) provided that the maximum allowable number of steps namely as (Nstep =1000). According to this basis the simulation program estimates error summation (which is constrained to be in no more than 0.1) and iteration number throughout each step which are required to reach a steady state condition. This is shown in Figs.(6,7 and 8).

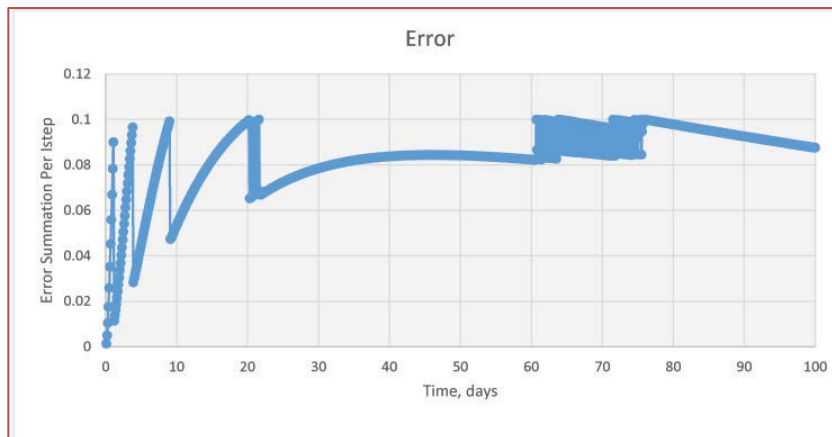


Fig.(6) Error Summation per Time step

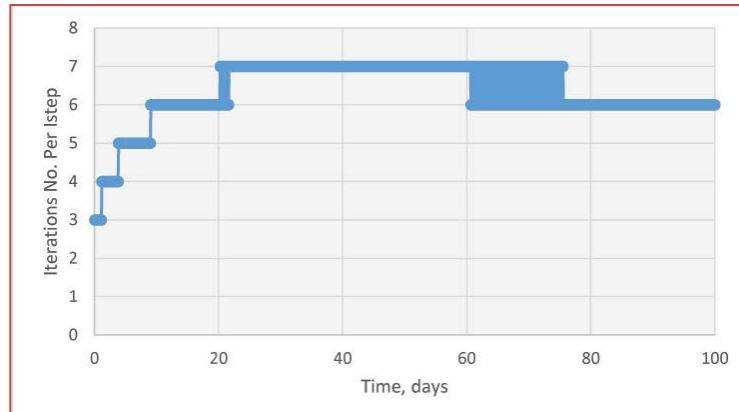


Fig.(7) Iteration No. per each Time Step

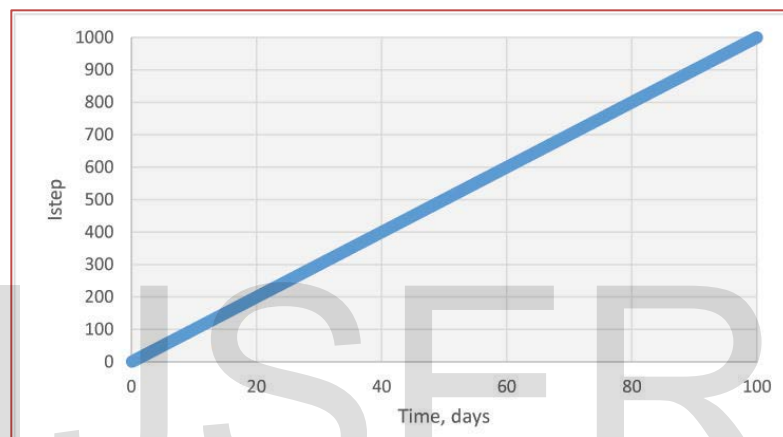


Fig.(8) Time Step per day

Contaminant Distribution & Natural Degradation

Al Hindyia city is costumed to dispose a (5liters) of waste water into Euphrates River near Al Hindyia Bridge (as indicated at Fig.(8)). The waste water represents a dangerous thread to human health and fishery survival since it comprises the followings:-

- 1- Pathogens such as bacteria, viruses and non-pathogenic bacteria.
- 2- Organic particles such food, feces, paper fibers, plant tissues ...etc.
- 3- Soluble organics such fruit sugars, proteins, drugs, oils ...etc.
- 4- Inorganic particles such as suspension materials, metals, ceramics...etc.
- 5- Soluble inorganics such as ammonia, salt, cyanide, hydrogen sulfide ...etc.
- 6- Animals such as protozoa, insects ... etc.
- 7- Gases such as hydrogen sulfide, carbon dioxide, methane ...etc.
- 8- Emulsions such as paints, adhesives, hair colorants... etc.
- 9- Toxics such as pesticides, poisons... etc.
- 10- Hazardous substances such as Pharmaceuticals and hormones.

Laboratorial tests indicate that a waste water of 545mg/liters disposing continuously into Euphrates River near the bridge. The simulated and laboratorial results that a natural degradations of pollutant have been occurred to produces a zero concentration at 555m D/S of the disposing point as shown in Fig.(9).

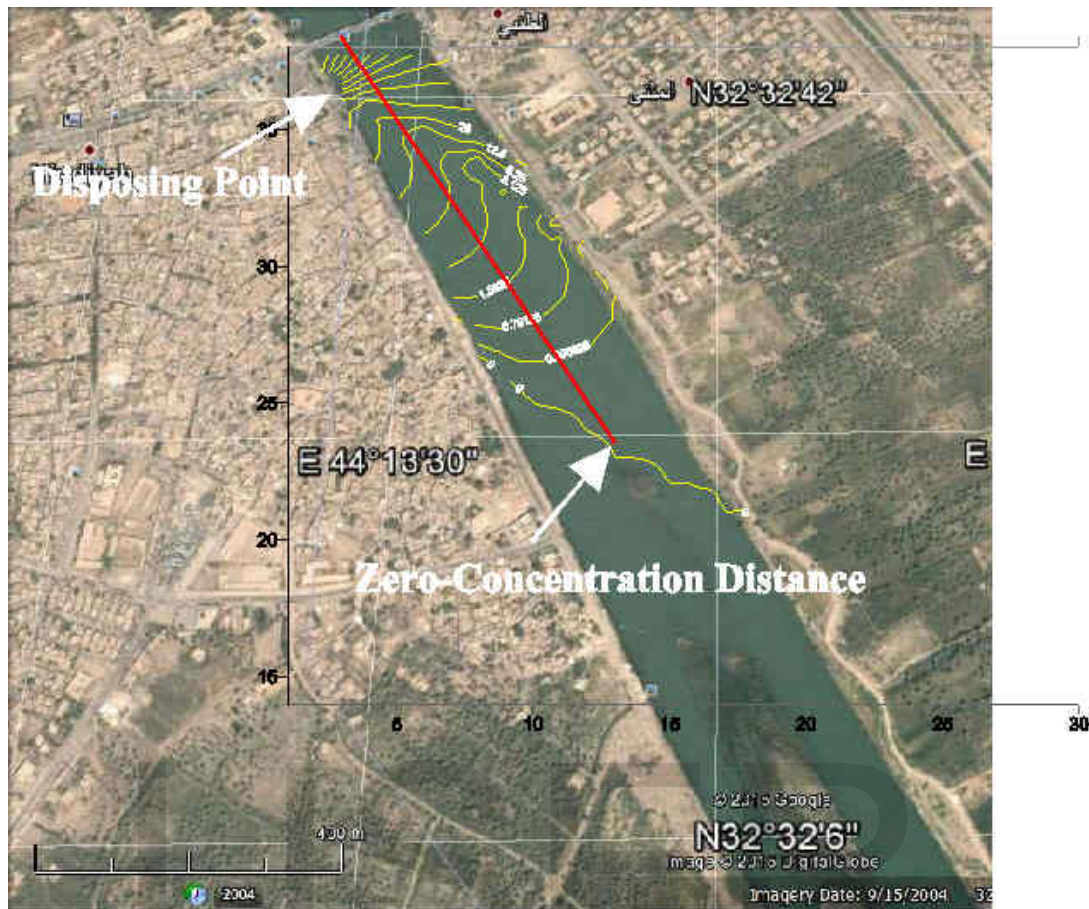


Fig.(9) Natural and Simulated Distribution of Pollutants along the River

Other Pollutants Diffusion in Euphrates

A huge number of harmful and toxic menials (organics or inorganics) are disposing continuously into the Euphrates River and then water is reused again for urban purposes. The diffusion of each element depends mainly on several factors namely as; diffusion coefficient, pollutant discharge, and concentration. Each element is characterized with a certain diffusion coefficient through water as outlined by many researchers as Chapra (1997) and martin (2000). It a good application is to locate the station D/S of the disposing point at which the element concentration becomes zero due to natural purification and degradation. Many values for diffusion coefficient are entered in the model to locate a zero-concentration distance D/S the disposing point (zero-concentration point: is the point at which the water can be used for urban purposes). The model is run for long period. Fig.(10) shows the relationship between diffusion coefficient chemicals and zero-concentration distance.

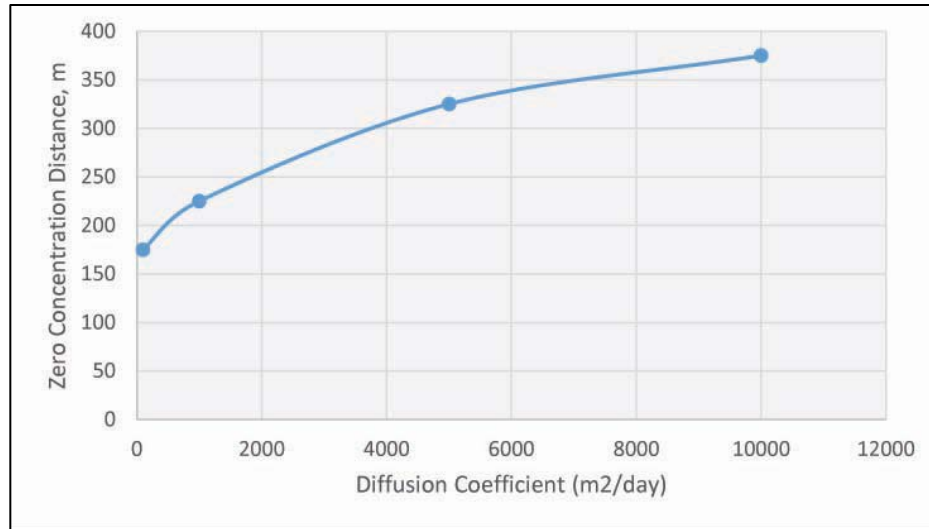


Fig.(10) Zero- Concentration Distance Versus Diffusion Coefficients

Conclusions:

It is concluded that:-

- 1- The zero-concentration distance is met at 555m D/S of disposing point of waste water into Euphrates River due to natural purification, dilution and degradation.
- 2- the model is efficient and flexible in dealing with the input and output data

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C Basic Contaminant Transport of Surface
Water Runoff Simulation Program
C By Dr. Najah M. Lateef Al Maimuri, 2015
C*****
REAL SA, SB, SC, SD, SE, DELTA, NR, NC,
V,ERROR,SQ, V, D, Nstep, S, SCO
PARAMETER (IMAX=40,JMAX=40)
DIMENSION
AA(IMAX,JMAX),BB(IMAX,JMAX),CC(IMAX,J
MAX),DD(IMAX,JMAX),EE(IMAX,JMAX),SAMI
R(IMAX,JMAX),Q(IMAX,JMAX),
1CO(IMAX,JMAX),C(IMAX,JMAX)
C Where: NR: Number of Matrix Rows, NC:
Number of Columnsm V: Surface Flow
Velocity in(m/sec)
C Delta: Mesh Dimension in(m), D: Difusion
Factor in (m2/day), Error: max Allowable
Error pre one Time step
C Nsteps: Max Time Steps
C Data Files
OPEN(1,FILE='AA.DAT')
OPEN(2,FILE='BB.DAT')
OPEN(3,FILE='DD.DAT')
OPEN(4,FILE='EE.DAT')
OPEN(5,FILE='QQ.DAT')
OPEN(6,FILE='INCO.DAT')
OPEN(7,FILE='SAMIR.DAT')
OPEN(8,FILE='MODEL.DAT')
OPEN(9,FILE='MODELworksheet.DAT')
OPEN(10,FILE='ERROR.DAT')
C Where A(i,j)=(1+(v(i,j)*Delta)/2D),BB(i,j)
=(1-(v(i,j)*Delta)/2D)
C DD=1, EE=1, QQ: Contaminant Discharge
m3/day, INCO: Initial Concentration
C Samir: (0101) file, Model: Output Data
file
DO 10 J=1,NR
READ(7,5)I,(SAMIR(I,J),I=1,NC)
5 FORMAT(I2,4X,30F5.1)
10 CONTINUE
C Reading Data Files
C*****
DO 45 I=1,NR
READ(1,15)I,(AA(I,J),J=1,NC)
READ(2,15)I,(BB(I,J),J=1,NC)
READ(3,15)I,(DD(I,J),J=1,NC)
READ(4,15)I,(EE(I,J),J=1,NC)
READ(5,17)I,(Q(I,J),J=1,NC)
READ(6,16)I,(CO(I,J),J=1,NC)
15 FORMAT(I2,4X,30F7.3)
16 FORMAT(I2,4X,30F7.1)
17 FORMAT(I2,4X,30F10.1)
45 CONTINUE
C DO 31 I=1,NR
C DO 31 J=1,NC
C REP=CO(I,J)

C Cold(I,J)=REP
C31 CONTINUE
C Estimation of New Concentrations
C*****
DT=0
C Where DT: TIME INCREMENT, Iter: is
the Iteration Number
TIME=0
DO 500 ISTEP=1,NSTEPS
DT=DT+1
ITER=1
19 E=0
C SEARCHING FOR NEW ONCENTRATIONS
C*****
DO 25 I=1,NR
DO 25 J=1,NC
S=SAMIR(I,J)
IF(S.EQ.0)GOTO 25
CC(I,J)=-(4+((V*Delta)/(D*DT)))
SC=CC(I,J)
SCO=(CO(I,J)/DT)+Q(I,J)
SQ=DELTA**2/D
SDC=-{SCO*SQ}
C G ARRAYS CALCULATION
SA=AA(I,J), SB=BB(I,J), SD=DD(I,J),
SE=EE(I,J)
GDC=SDC/SC, GA=SA/SC, GB=SB/SC,
GD=SD/SC
GE=SE/SC
C(I,J)=GDC-GA*C(I,J-1)-GB*C(I,J+1)-
GD*C(I-1,J)-GE*C(I+1,J)
C Cnew(I,J)=C(I,J)
E=E+ABS(Co(I,J)-C(I,J))
25 CONTINUE
IF(E.LT.ERROR)GOTO 33
34 DO 30 I=1,NR
DO 30 J=1,NC
Co(I,J)=C(I,J)
30 CONTINUE
ITER=ITER+1
GOTO 19
33 write(10,*)DT,ISTEP,ITER,E
IF(ISTEP.GT.NSTEPS)GOTO 125
C WRITE(*,*)c(11,13),DT
500 CONTINUE
C Output Data
125 DO 110 I=1,NR
DO 110 J=1,NC
S=SAMIR(I,J)
IF(S.EQ.0)GOTO 110
WRITE(9,*)J,NR+1-I,C(I,J)
110 CONTINUE
end
    
```

Fig.(5) Basic Contaminant Transport Simulation Program